Invited Lecture

Language and Learning Mathematics: A Sociocultural Approach to Academic Literacy in Mathematics

Judit. N. Moschkovich[1]

ABSTRACT This invited lecture summarized my work on language and learning mathematics. I described a theoretical framework for academic literacy in mathematics (Moschkovich, 2015a, 2015b) that can be used to analyze student contributions and design lessons. The presentation included a classroom example and recommendations for instruction that integrates attention to language. Although the example is from a bilingual classroom, the theoretical framing and the recommendations are relevant to all mathematics learners, including monolingual students learning to communicate mathematically.

Keywords: Language; Learning; Sociocultural.

1. Introduction

This talk summarized a sociocultural framework for academic literacy in mathematics (Moschkovich, 2015a, 2015b) that uses a complex view of both mathematics and language, focuses on understanding (not computation), and emphasizes mathematical practices (Moschkovich, 2013a). To support all students in learning mathematics we need to shift from simplified views of mathematical language as single words to a broader definition of academic literacy — not just learning words but learning to communicate mathematically. Mathematics instruction must shift from focusing on low-level language skills (i.e., vocabulary or single words) to using an expanded definition of academic literacy in mathematics that includes mathematical practices and discourse. This sociocultural framework can be used to uncover how students participate in mathematical practices, hear how language provides hybrid resources for mathematical activity, and design lessons that include attention to language.

1.1. Why integrate language into research on learning mathematics?

In the talk, I described how I integrated language into research on mathematics learning and teaching. I first summarized a theoretical framework that is a socio-cultural approach to "academic literacy in mathematics." I then used a classroom example to illustrate that theoretical framework and make recommendations for instruction.

This integration was motivated by theorical and practical goals. Integrating language into research on mathematics learning provides a fuller theoretical account

[1] University of California-Santa Cruz, Santa Cruz, CA 95064, USA. E-mail: jmoschko@ucsc.edu

of mathematical thinking that includes language(s). Integrating research on learners who use more than one language into research on mathematics learning/teaching is important not only to include this student population, but also because language is central to a full theorical account. Research with bilingual learners provides a window on language: while language can be invisible in monolingual situations, a setting with bilingual learners makes language visible. In this way, bilingual learners are a gift.

There are also practical reasons for this integration. Clearly, this integration is crucial to improve instruction for students who are learning the language of instruction (LOI) and are bilingual/multilingual. Nevertheless, integrating language is important not just for those student populations imagined to have issues with language, it is important for all students. Mathematics is a discipline associated with power and authority; power and authority are enacted through and mediated by language. Integrating language is particularly important for students from communities with a history of lack of opportunities to learn mathematics due to imagined issues with language. However, this integration is important for all students because all students experience power and authority through language practices in mathematics classrooms.

1.2. A little history: themes in my research

I started out looking closely at student conceptions of linear functions. I examined how students understand and use the connections between equations and their graphs. I was especially interested in how a discussion with a peer supported learning about equations and lines (Moschkovich, 1996). This led me to thinking about the mathematics register and mathematical discourse (Moschkovich, 2002, 2007a). I worked with bilingual students and this led me to exploring the role of language in learning mathematics, documenting how bilingual students communicate mathematically (Moschkovich, 1999, 2007b). Moschkovich (2002) was my first attempt to understand and use the concept of register. I also aimed to shift from a view of language as an obstacles to a resource, what I would now call from a deficit to an asset view of bilingual learners.

I used Vygotskian and neo-Vygotskian theories of learning (i.e., Forman, 1996; Vygotsky, 1978, 1979, 1987) to frame my research. One goal was to reconcile my theoretical commitments to Vygotskian perspectives with more cognitive views of mathematical thinking. I initially struggled to answer several (fundamentally Vygotskian) questions, especially where to see mediation by social-cultural artifacts. I came to see that, for me, the answer to this question lay in clarifying the category of mathematical practices, connecting those to mathematical discourse, and using appropriation to describe how learners learn to use those practices.

Using Vygotskian theories and work in sociolinguistics (e.g., Gee, 1990), I analyzed discussions of mathematical problems among students or between a learner and an adult. My analyses have focused on identifying and describing central aspects of mathematical practices. In one article, “Appropriating mathematical practices: A case study of learning to use and explore functions through interaction with a tutor,” I returned to the theme of mathematical discourse and added an analysis of mathematical practices (Moschkovich, 2004). As I realized that the construct "mathematical practices" was central to my work, I wrote a chapter titled "Issues regarding the concept of mathematical practices (Moschkovich, 2013).

More recently, starting in 2015, I integrated these themes — student thinking, language, discourse, and practices — into a theoretical framework that I call "academic literacy in mathematics (Moschkovich, 2015a, 2015b)." Here I summarize that framework and briefly define the three components of academic literacy in mathematics: mathematical proficiency, mathematical practices, and mathematical discourse. I have used that framework to analyze classroom discussions (Moschkovich, 2015a) and describe how a teacher provided scaffolding for mathematical practices (Moschkovich, 2015c).

1.3. Theoretical framing and assumptions

The framework draws on sociocultural and situated perspectives of learning mathematics (Brown et al., 1989; Greeno, 1998) as a discursive activity (Forman, 1996) that involves participating in a community of practice (Lave and Wenger, 1991), and using multiple material, linguistic, and social resources (Greeno, 1998). Mathematical activity is assumed to involve not only individual mathematical knowledge but also collective mathematical practices and discourses.

A sociocultural perspective brings several assumptions to defining academic literacy in mathematics. The first assumption is that mathematical activity is not merely cognitive or individual; instead, it is simultaneously cognitive, social, and cultural. Second, the focus is on the potential for progress in what learners say and do, not on learner deficiencies. The third assumption is that participants bring multiple perspectives to any situation and that meaning is not static but situated: representations and utterances have multiple meanings; meanings for words (or inscriptions) are situated, constructed while participating in practices, and negotiated through interaction (Moschkovich, 2008).

A sociocultural perspective of academic literacy in mathematics[2] provides a complex view of mathematical proficiency as participation in discipline-based practices that involve reasoning, understanding, and communicating. A situated and sociocultural perspective on bilingual mathematics learners (Moschkovich, 2002) shifted the focus from looking for deficits to identifying the mathematical discourse practices evident in student contributions (e.g., Moschkovich, 1999). The sociocultural

[2] This sociocultural perspective builds on previous work where I described a sociocultural view of mathematics learners who are bilingual and/or learning English (Moschkovich, 2002), of mathematical discourse (Moschkovich, 2007a), and of mathematical practices (Moschkovich, 2013). In other publications (e.g., Moschkovich, 2008), I described how mathematical discourse is situated, involves coordinated utterances and focus of attention, and combines everyday and academic registers. The definition of academic literacy in mathematics used here brought together and built on different aspects of those analyses.

perspective in Moschkovich (2002, 2004, 2007a) also provided a theoretical framework for recognizing the mathematical practices in student contributions.

2. Academic Literacy in Mathematics

Academic literacy in mathematics includes three integrated components: mathematical proficiency, mathematical practices, and mathematical discourse. These three components are intertwined, should not be separated during instruction, and cannot be separated when analyzing student mathematical activity or designing mathematics lessons.

The view of academic literacy in mathematics presented here is different than previous approaches to academic language in several ways. First, the definition includes not only cognitive aspects of mathematical activity — what happens in one's mind, such as mathematical reasoning, thinking, concepts, and metacognition – but also social and cultural aspects — what happens with other people, such as participation in mathematical practices — and discourse aspects — what happens when using language (reading, writing, listening, or talking about mathematics). Most importantly, the components of academic literacy in mathematics work together, not separating mathematical language from proficiency or practices.

This definition goes beyond narrow views of mathematical language as vocabulary, definitions, or formal language because these views limit learners' access to high-quality instruction. A focus on single words or vocabulary limits access to complex texts and high-level mathematical ideas and to opportunities for students to understand and make sense of those texts. The assumption that meanings are static and given by definitions limits students' opportunities to make sense of mathematics texts for themselves. The assumption that mathematical ideas should always and only be communicated using formal language limits the resources that students can use to communicate mathematically, excluding or dismissing resources such as informal, everyday, or home language(s) that have been documented as important for communicating mathematically.

In contrast, the view of mathematical language used here assumes that meanings for academic language are situated and grounded in the mathematical activity that students are actively engaged in. For example, meanings for the words in a word problem do not come from the definition in a word list provided by the teacher. Instead, students negotiate meanings as they work on a problem, communicate with peers, and develop their solutions. A complex view of mathematical language also means that lessons must include multiple modes (not only reading and talking but also listening and writing), multiple representations (gestures, objects, drawings, tables, graphs, symbols, etc.), and multiple ways of using language (formal school mathematical language, home languages, and everyday language). In addition, this definition ensures that academic literacy in mathematics goes beyond simplified views of mathematics as computation. It includes the full spectrum of mathematical proficiency, balancing procedural fluency with conceptual understanding; it also includes mathematical practices and emphasizes student participation in discourse practices.

2.1. Defining academic literacy in mathematics

Academic literacy in mathematics is more complex than simply combining alphabetic literacy with proficiency in mathematics. Reading and solving a word problem entails not only proficiency in mathematics but also competencies in using mathematical practices and discourses. Typically, "literacy" is interpreted as referring to words and "mathematics" as referring to numbers. For example, we could imagine that solving the word problem below involves "literacy" in the activity of reading and understanding the words, and "mathematics" in the activity extracting the numbers and relating them through arithmetic operations:

> Jane, Maria, and Ben each have a collection of marbles. Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles. Find how many marbles Maria has.

However, reading and solving this word problem entails not only mathematical proficiency, proficiency in the content of mathematics, but also competencies in using mathematical practices such as making sense of the problem. If students are asked to communicate their solutions to a peer or to the whole class, then solving this word problem also involves mathematical discourse, communicating one's thinking and describing one's solution. These three components cannot be separated when considering mathematical tasks, analyzing student mathematical activity, or designing mathematics instruction.

The complexity of academic literacy in mathematics is evident in another word problem:

> A boat in a river with a current of 3 mph can travel 16 miles downstream in the same amount of time it can go 10 miles upstream. Find the speed of the boat in still water.

Solving this word problem requires much more than reading and understanding the text and then deciding what arithmetic operations to use or what equation to write. One surely cannot solve this word problem by using key mathematics vocabulary words to extract the correct numbers (or quantities or variables), relate them using the correct arithmetic operation, or write the correct equation. Instead, what is required are all three components simultaneously: mathematical proficiency, mathematical practices, and mathematical discourse. Possible mathematical practices needed for this problem include modeling a situation (as one carefully imagines what is going on in the situation) and using or connecting representations (if one draws a picture). This word problem also illustrates how academic literacy in mathematics is not principally about technical vocabulary. The crucial vocabulary for understanding this problem situation is not mathematical or technical vocabulary. Instead, the challenging vocabulary might be upstream, downstream, and "in still water."

Simplified views of academic language in mathematics focus on words, assume that meanings are static and given by definitions, separate language from mathematical knowledge and practices, and limit mathematical discourse to formal language. In contrast, academic literacy in mathematics as defined here includes three integrated components: mathematical proficiency, mathematical practices, and mathematical discourse. This view of academic literacy in mathematics is different than previous approaches to academic language in several ways. First, the definition includes not only cognitive aspects of mathematical activity — such as mathematical reasoning, thinking, concepts, and metacognition — but also sociocultural aspects — participation in mathematical practices — and discursive aspects — participation in mathematical discourse.

This is an integrated view of three components working in unison, rather than isolating academic language from mathematical proficiency or mathematical practices. Second, this integrated view, rather than separating academic language from mathematical proficiency or practices, views the three components as working in unison. Separating language from mathematical thinking and practices can have dire consequences for students. This separation can make students seem more deficient than they are, since they may express their mathematical ideas through imperfect language, but may still be engaged in correct mathematical thinking, and they may participate in mathematical practices through other modes, for example using objects, drawings, or gestures to show a result, describe regularity in data, or illustrate a mathematical concept. Lastly, this definition includes the full spectrum of mathematical proficiency, balancing fluency in computing with an emphasis on conceptual understanding, reasoning, and communicating.

2.1.1. Defining mathematical proficiency

A description of mathematical proficiency (Kilpatrick et al., 2001) shows five intertwined strands: Conceptual understanding, Procedural fluency, Strategic competence, Adaptive reasoning; and Productive disposition. Procedural fluency is knowing how to compute. Conceptual understanding is fundamentally about the meanings that learners construct for mathematical solutions: knowing the meaning of a result (what the number, solution, or result represents), knowing why a procedure works, and explaining why a particular result is the right answer. Reasoning, logical thought, explanation, and justification are closely related to conceptual understanding. Student reasoning is evidence of conceptual understanding when a student explains why a particular result is the right answer or justifies a conclusion. The five strands of mathematical proficiency provide a cognitive account of mathematical activity focused on knowledge, metacognition, and beliefs. However, from a sociocultural perspective, mathematics students are not only acquiring mathematical knowledge, they are also learning to participate in valued mathematical practices (Moschkovich, 2004, 2013a).

2.1.2. Defining mathematical practices

The term practice shifts from purely cognitive accounts of mathematical activity to assuming the social, cultural, and discursive nature of doing mathematics. I use the terms practices drawing on Scribner's (1984, p. 13) practice account of literacy to "highlight the culturally organized nature of significant literacy activities and their conceptual kinship to other culturally organized activities involving different technologies and symbol systems". This definition implies that mathematical practices are culturally organized, involve symbol systems, and are related conceptually to other mathematical practices. From this perspective, mathematical practices are not only cognitive — i.e., involving mathematical thinking and reasoning — but also social and cultural — arising from communities and marking membership in communities — and semiotic — involving semiotic systems (signs, tools, and their meanings).

Academic mathematical practices can be understood in general as using language and other symbols systems to think, talk, and participate in the practices that are the objective of school learning. There is no single set of mathematical practices or one mathematical community; practices vary across communities of research mathematicians, traditional classrooms, and reformed classrooms. However, across these various communities, there are common practices that can be labeled as academic mathematical practices. Examples of mathematical practices include problem solving, sense-making, reasoning, modeling, abstracting, generalizing, using or connecting mathematical representations, imagining, and looking for patterns, structure, or regularity.

2.1.3. Defining mathematical discourse

A sociocultural framing of mathematical practices connects practices to discourse . In particular, discourse is central to participation in many mathematical practices, and meanings for words are situated and constructed while participating in mathematical practices. Academic mathematical discourse has been described as having some general characteristics. In general, particular modes of argument, such as precision, brevity, and logical coherence, are valued (Forman, 1996). Abstracting, generalizing, and searching for certainty are also highly valued. Generalizing is reflected in common mathematical statements, such as "The angles of any triangle add up to 180 degrees", "Parallel lines never meet", or "a + b (always) equals b + a". What makes a claim mathematical is, in part, the detail in describing when the claim applies and when it does not. Mathematical claims apply only to a precisely and explicitly defined set of situations and are often tied to mathematical representations (symbols, graphs, tables, or diagrams). Many valued academic mathematical practices involve mathematical discourse.

The academic literacy in mathematics framework goes beyond low-level language skills, using a view of mathematical discourse that includes multiple modes, symbol systems, registers, and languages. Mathematical discourse is assumed to be multi-modal and multi-semiotic (using multiple sign systems). Meanings are not provided by static dictionary definitions, but situated in local history, practices, and socio-cultural context. Mathematical discourse draws on hybrid resources; during classroom mathematical discussions students use both everyday and formal mathematics registers.

I use the phrase mathematical discourse because there are multiple meanings for "language" and to emphasize that discourse is much more than language. I do not use the phrase "academic language" because it can be reduced to single words, vocabulary, or grammar. In contrast, I use a view of mathematical discourse not as a list of words with precise meanings but the communicative competence (Hymes, 1972/2009) necessary and sufficient for competent participation in mathematical practices. Work on the language of disciplines (e.g., Pimm, 1987) provides a complex view of mathematical language as not only specialized vocabulary — new words and new meanings for familiar words — but also as extended discourse that includes syntax and organization, the mathematics register (Halliday, 1978), and discourse practices (Moschkovich, 2007a). The mathematics register includes styles of meaning, modes of argument, and mathematical practices and has several dimensions such as the concepts involved, how mathematical discourse positions students, and how mathematics texts are organized.

3. A Classroom Example of Mathematical Practices

If students are participating in academic literacy in mathematics, we can see and hear them actively participating in mathematical practices, many of which are discursive. This classroom example illustrates using the framework to uncover how students use mathematical practices and hybrid language practices (Gutierrez et al, 1999) to participate in a mathematical discussion.

The lesson excerpt comes from a third-grade bilingual Spanish-English classroom in an urban California school. In this classroom, there were thirty-three students . In general, this teacher introduced students to topics in Spanish and then later conducted lessons in English. The students had been working on a unit on two-dimensional geometric figures. For several weeks, instruction had included vocabulary such as the names and properties of different quadrilaterals in both Spanish and English. Students had been talking about shapes and the teacher had asked them to point, touch, and identify different quadrilaterals. The teacher identified this lesson as a lesson where students would be using English to discuss different shapes.

Below is an excerpt from the transcript for this lesson involving descriptions of a rectangle. (Brackets indicate transcript annotations.)

1. Teacher: Let's see how much we remembered from Monday. Hold up your rectangles. ... high as you can. [students hold up rectangles] Good, now. Who can describe a rectangle (for me)? Eric, can you describe it? [a rectangle] Can you tell me about it?
2. Eric: A rectangle has ... two ... short sides, and two ... long sides.

3. Teacher: Two short sides and two long sides. Can somebody tell me something else about this rectangle? If somebody didn't know what it looked like, what, what ... how would you say it?
4. Julian: Parallel(a). [holding up a rectangle]
5. Teacher: It's parallel. Very interesting word. Parallel, wow! Pretty interesting word, isn't it? Parallel. Can you describe what that is?
6. Julian: Never get together. They never get together [runs his finger over the top length of the rectangle].
7. Teacher: OK, what never gets together?
8. Julian: The parallela ... they ... when they, they get, they go, they go higher [runs two fingers parallel to each other first along the top and base of the rectangle and then continues along those lines] they never get together.
9. Antonio: Yeah!
10. Teacher: Very interesting. The rectangle then has sides that will never meet [runs fingers along top and base of an invisible rectangle] those sides will be parallel [motions fingers vertically in parallel lines]. Good work. Excellent work.

3.1. Uncovering mathematical practices

One recommendation for instruction is to use this framework to focus on mathematical practices, not "language" as words, vocabulary, or formal definitions. An overemphasis on correct vocabulary and formal language limits the linguistic resources for learning/teaching math with conceptual understanding and precludes students from participating in valued mathematical practices.

What mathematical practices did Julian use? There were several mathematical practices evident in Julian's original utterance in line 8. Julian was abstracting and generalizing. He was describing an abstract property of parallel lines and making a generalization saying that parallel lines will never meet. He was also imagining what happens when the parallel sides of a rectangle are extended. If we focused only on whether he did or did not use mathematical vocabulary, we would miss Julian's use of these important mathematical practices.

Emerging language and ideas are imperfect and may be difficult to understand. In this example, uncovering the mathematical practices in Julian's contributions is challenging. Julian's utterances in turns 4, 6, and 8 are difficult to hear and interpret. He said the word "parallela" with hesitation. His voice trailed off, so it is difficult to know whether he said "parallelo" or "parallela." His pronunciation could be interpreted as a mixture of English and Spanish; the "ll" sound pronounced in English and the addition of the "o" or "a" pronounced in Spanish. Was this hesitation due to issues with the pronunciation or the mathematical idea? It is impossible to answer this question! Instead, the framework can be used to focus on mathematical practices and to notice that Julian accurately described a property of parallel lines. If we focus only on formal vocabulary, we miss the mathematical practices. Instead of focusing on single words or formal vocabulary, it is more important to listen for the meaning of the whole utterance, since that is the way to uncover mathematical practices.

3.2. Uncovering hybrid language practices

The second recommendation is to use the academic literacy in mathematics framework to treat everyday discourses as resources. Everyday and home registers have been documented as providing resources for communicating mathematically. Students are likely to use hybrid language practices that combine everyday and formal registers.

What language resources did Julian use to communicate his mathematical ideas? Julian's pronunciation in turns 4 and 8 is an example of a hybrid language practice. His utterances can be interpreted as a mixture of English and Spanish, the word "parallel" pronounced in English, and the added "a" pronounced in Spanish. In Spanish, the word parallel would agree with the noun (line or lines), in both number (plural or singular) and gender (masculine or feminine; "parallel lines" translates to "líneas paralelas," "parallel sides" translates to "lados paralelos"). The grammatical structure in turn 8 can also be interpreted as a mixture of Spanish and English. The apparently singular "parallela" in turn 8 was preceded by the word "the" (which can be either plural or singular) and then followed by a plural "when they go higher."

Julian also used colloquial expressions such as "go higher" and "get together" rather than the formal terms "extended" or "meet." These everyday expressions were not obstacles but resources to communicate a mathematical idea. These phrases are instances of everyday phrases used with mathematical meaning. Julian used hybrid language resources that drew on both everyday and academic registers. He did not use technical phrases but an everyday phrase with mathematical meaning. The discussion was mathematical not because it involved technical mathematical terms, but because it involved mathematical concepts and practices. This example illustrates how the everyday and academic registers are not in opposition and how both can provide resources to communicate mathematical ideas.

3.3. Teacher moves

The excerpt also illustrates how this teacher, rather than requiring students to use an idealized version of perfect language, accepted and built on students' hybrid use of language to support student participation in a mathematical discussion. The teacher used several teacher moves such as asking for clarification, probing what students mean, and revoicing student statements.

Revoicing is an important way teachers can build on students' own use of mathematical practices or add new mathematical practices to a discussion. In turn 5, the teacher accepted Julian's response, revoicing it as "It's parallel," and probed what Julian meant by "parallela." In turn 10, the teacher revoiced Julian's contribution in turn 8: "the parallela, they" became "sides," and "they never get together" became "will never meet, will be parallel."

In this case, the teacher's revoicing made Julian's claim more precise, introducing a new mathematical practice, attending to the precision of a claim. In line 10, the teacher's claim is more precise than Julian's claim because the second claim refers to the sides of a quadrilateral, rather than any two parallel lines. Revoicing also provided opportunities for students to hear more formal mathematical language. The teacher revoiced Julian's everyday phrase "get together" as "meet" and "will be parallel." Revoicing can be used to scaffold mathematical practices and use formal language (Moschkovich, 2015c).

4. How Instruction can Focus on Academic Literacy in Mathematics

The view of academic literacy in mathematics described here integrates mathematical proficiency with mathematical practices and discourse. Separating language from mathematical proficiency limits learners' access to conceptual understanding. Separating language from mathematical practices curtails students' opportunities to participate in mathematical practices. Not allowing students to use informal language, typically acquired before more formal ways of talking, also limits the resources to communicate mathematically. Lastly, focusing on correct vocabulary curtails opportunities for students to express themselves mathematically in what are likely to be imperfect ways, especially as they are learning new ideas.

In contrast, the view of academic literacy in mathematics described here focuses on mathematical practices and uses and expanded view of language that includes informal language as a resource. Mathematics lessons that integrate language provide students opportunities to participate in mathematical practices, negotiate meanings, and use multiple discourses and registers. Teachers can support students as they negotiate meanings for mathematical language; this negotiation is best when it is grounded in students' own mathematical work, instead of giving students definitions separate from their mathematical activity (Moschkovich, 2015a, 2015b).

For students learning mathematics, informal language is important, especially when students are exploring a new mathematical concept or discussing a problem in small groups. By learning to recognize how learners actively use hybrid language practices to engage in understanding, reasoning, and communicating, teachers can provide opportunities for students to participate in all three components of academic literacy in mathematics. Students can use informal language during exploratory talk (Barnes, 2008) or when working in a small group (Herbel-Eisenmann et al., 2013). Such informal language reflects important mathematical thinking (for examples, see Moschkovich, 1996, 2008). In other situations, for example, when presenting a solution or writing an account of a solution, using more formal academic mathematical language becomes more important.

Mathematics instruction needs to shift from simplified views of language as vocabulary and carefully consider when and how to emphasize correct vocabulary and formal language. Such views severely limit the linguistic resources teachers and students can use to teach and learn mathematics, and separate language from mathematical practices. Focusing instruction on vocabulary limits students' access to the five strands of mathematical proficiency and curtails students' opportunities to participate in mathematical practices. We must leave behind simplified views of language as vocabulary, embrace the multimodal and multi-semiotic nature of mathematical activity, and shift from monolithic views of mathematical talk or dichotomized views of everyday and mathematics registers (Moschkovich, 2010). An overemphasis on correct vocabulary and formal language limits the linguistic resources teachers and students can use in the classroom to learn mathematics with understanding.

The question is not whether students should learn vocabulary but rather when and how instruction can best support students as they learn not only the meanings of words and phrases but also how to participate in mathematical practices. Vocabulary drill, practice, definitions, or lists are not the most effective way to learn to communicate mathematically. Instead, vocabulary acquisition (whether it is in a first or second language) occurs most successfully in instructional contexts that are language-rich, actively involve students in using language, require both receptive and expressive understanding, and require students to use words in multiple ways over extended periods of time (Blachowicz and Fisher, 2000; Pressley, 2000). To develop oral and written communication, students need to participate in negotiating meanings (Savignon, 1991) and in tasks that require student output (Swain, 2001). Instruction should provide opportunities to actively use mathematical language to communicate about and negotiate meaning for mathematical situations.

Overall, dichotomies such as everyday/academic or formal/informal are not useful for research or practice (Moschkovich, 2010). Classroom discussions draw on hybrid resources from both academic and everyday contexts, and multiple registers co-exist in math classrooms. Everyday ways of talking should not be seen as obstacles to participation in academic mathematical discussions but as resources teachers can build on to support students in learning more formal mathematical ways of talking. Teachers need to hear the mathematical content in students' everyday language, build on that everyday language, and support or scaffold (Moschkovich, 2015c) more formal language. Everyday language is not only a starting place for learners; it supports reasoning, facilitates communication, and grounds meanings.

With a complex definition of academic literacy in mathematics, teachers can choose (or design) tasks that support academic literacy in mathematics, provide opportunities for learners to participate in academic literacy in mathematics, and recognize academic literacy in mathematics in student activity. When designing instruction, teachers can consider how each component of academic literacy in mathematics might appear and how to provide students opportunities to participate in each of the three components. If students are participating in academic literacy in mathematics as defined here, then we see or hear them engaged in the full spectrum of mathematical proficiency as they participate in mathematical practices, many of which are discursive.

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